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AMBCOM was found to give generally better agreement with the above data than did RADAR C. Comparison of details of model predictions from the two computer programs for the above data-base is used to form an understanding of this improvement in prediction capability.

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COMPARISON WITH OBLIQUE SOUNDER DATA OF HIGH-LATITUDE HF PROPAGATION
PREDICTIONS FROM "RADAR C" AND "AMBCOM" COMPUTER PROGRAMS

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ABSTRACT

A study is done using two HF propagation prediction programs - "RADAR C" and "AMBCOM" - to determine how well they predict median values of oblique sounder data of maximum observed frequencies (MOF) at high latitudes. The main differences between RADAR C and AMBCOM are the inclusion in the latter of high-latitude ionosphere and auroral absorption models, as well as a more sophisticated and accurate ray-tracing scheme. The data used for comparison are taken from Reference [1] for the Winnipeg-Resolute Bay path in the year 1959 (also discussed by Petrie and Warren [2]) and from Folkestad [3] for the Andoya-Ft. Monmouth and Andoya-College paths in 1964. The data for the Winnipeg-Resolute Bay corresponds to high sunspot number, while the others correspond to low sunspot number. Hence, this study provides information on the performance of the two programs for various high-latitude paths at both high and low sunspot number.

AMBCOM was found to give generally better agreement with the above data than did RADAR C. Comparison of details of model predictions from the two computer programs for the above data-base is used to form an understanding of this improvement in prediction capability.

INTRODUCTION

This paper begins with a summary of the differences between the basic ionospheric models and raytracing assumptions made in constructing the RADAR C and AMBCOM programs. User options selected for this study are discussed in Section 2. In Section 3, comparison of predictions from the two programs with available oblique sounder data is presented with appropriate explanation. In Section 4, the comparisons with data are discussed in terms of what they reveal about the significances of the differences between the two programs, and conclusions are formed regarding the apparent reasons for improved predictive capability of AMBCOM over RADAR C. In the final section, suggestions for directions in future work towards improving HF propagation prediction in high-latitude regions are made.

SECTION 1 - SOME BASIC ASSUMPTIONS USED IN THE DEVELOPMENT OF RADAR C
AND AMBCOM COMPUTER PROGRAMS

The developmental histories of RADAR C and AMBCOM are different, and this fact accounts for some of the differences between the two programs. RADAR C was developed to predict performance of over-the-horizon radars (Headrick, et. al. [4], Lucas, et. al. [5]). Thus, RADAR C has only a coverage option, not a point-to-point or "homing" option. The propagation model is based on virtual geometry and is essentially the same as that of ITS -78 (Barghausen, et. al. [6]) and IONCAP (Teters, et. al., [7]). AMBCOM was derived from the NUCOM program developed at SRI International (under the sponsorship of the Defense Atomic Support Agency, DASA, and its successor, the Defense Nuclear Agency, DNA). The purpose of NUCOM is to predict the effects of a nuclear disturbance on ionospheric communication channels (Nielson, et. al. [8]), and as a part of this objective, AMBCOM was developed to predict HF propagation in an undisturbed, or ambient, ionosphere. The raytracing scheme in NUCOM/AMBCOM was developed specifically to permit the treatment of a non-horizontally stratified ionosphere in the direction of propagation (i.e., it includes modelling of longitudinal, but not transverse, tilts, so that propagation is along the great circle path). AMBCOM has both coverage and point-to-point options.

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Table 1, extracted from an AMBCOM user's manual prepared by SRI International [9], highlights some important differences between the two programs. Evidently, the two programs differ in several features relevant to prediction of MUF's, as discussed below.

TABLE 1 - SUMMARY OF SIGNIFICANT MODEL DIFFERENCES BETWEEN AMBCOM AND RADAR C		
MODELS	RADAR C	AMBCOM
<u>IONOSPHERE GENERATION</u>		
Deck (for coefficients)	"ESSARED"	"ESSABLU", WITH HIGH-LATITUDE MODIFICATIONS (Hatfield [10])
Median models	YES	YES - including auroral ionosphere
Spatial representation	4 samples	41 samples
Real-data input	Ionograms at 4 locations	9 parabolic parameters at up to 41 locations
<u>PROPAGATION MODEL</u>		
Raytracing method	Martyn's theorem	Semianalytic raytrace
Tilts, horizontal gradients	NO	YES
Topside reflections	NO	YES
Radar propagation	YES	YES
Point-to-point propagation	NO	YES

(1) Determination of ionospheric parameters is done using the so-called ESSA "blue deck" coefficients in AMBCOM with high-latitude modifications introduced by SRI International (Hatfield [10]). RADAR C uses the unmodified ESSA "red deck" coefficients.

(2) AMBCOM chooses up to 41 control points (depending on path length) to determine local ionospheric parameters such as critical frequencies, whereas RADAR C has a maximum of 4 control points available for the user to input.

(3) AMBCOM models the ionosphere with three parabolic layers of electron density as a function of height, and uses a semi-analytic, two-dimensional raytracing scheme based on a method due to Kift and Fooks (Nielson [11]). The physical bases of this scheme are the geometric optics solution to the wave equation and Fermat's principle of minimum phase (Kelso [12]). RADAR C uses vertical ionograms computed from a similar ionospheric model as AMBCOM, and converts to oblique propagation using Martyn's theorem (Davies, [13]). The scheme used in AMBCOM permits consideration of continuous ionospheric gradients along the direction of propagation, whereas the RADAR C scheme assumes horizontal stratification of the ionosphere at each reflection point.

An added difference, not explicitly noted in Table 1, is the fact that AMBCOM is better capable than RADAR C of considering composite modes involving reflections from the E, F1, and F2 layers, including topside reflections off of the lower layers (M-modes), as well as chordal or perigee modes (i.e., rays which do not intersect the earth between layer reflections), as possible modes of propagation. This improved capability of AMBCOM is due to its more accurate raytrace method, (e. g., in AMBCOM it is not assumed that the angle of incidence to a layer equals the angle of reflection). Although RADAR C is also capable of considering composite modes, the assumption of horizontal stratification prevents the consideration of tilts and chordal modes by this program, as a result of which the majority of modes found by RADAR C turn out to be simple modes (all reflections being off of the same layer). In summary, the treatment of modes in AMBCOM is closer to physical reality than that in RADAR C.

It should be noted here that neither of the programs is designed expressly for predicting maximum usable frequency (MUF) for a given model ionosphere, so that the program output has to be interpreted to estimate a MUF. For this study, since RADAR C does not have a point-to-point option, its output for a given condition is scanned for the maximum frequency whose ground range (for some takeoff angle) bracket the receiver, this being interpreted as the MUF. In interpreting AMBCOM output, the point-to-point option is chosen, and it is assumed that all modes which reach the ground within 100 kilometers ground distance of the receiver, or all chordal modes which reach less than 90 kilometers height above the receiver are detectable modes. The

ranges in these acceptance criteria are somewhat broader than normally used (20 km height being a more common limit for chordal modes, for example), but are believed to be representative of the range of distances from which modes can be detected, considering the accuracy of the ionospheric model and raytracing scheme, and broadening of the beam. The results of this study, judged by examining the output, are not highly sensitive to the choice of the above numbers.

SECTION 2 - USER OPTIONS TAKEN IN PERFORMING THE STUDY

Some of the user options available in the two programs are of relevance to this study, hence are discussed below:

(1) Four control points are used in this set of RADAR C runs, approximately uniformly spaced on the great circle path between transmitter and receiver.

(2) Sporadic E modes are not considered in this study (IOPES = 0 in AMBCOM, MAXMOD = 1 in RADAR C).

(3) 12 month running averages of monthly median sunspot number are used. Monthly median values of magnetic index K_p are used in AMBCOM. (RADAR C does not use K_p).

(4) In order to minimize the amount of computing time without a great sacrifice in accuracy, only integer values of frequencies in the range of 1 - 30 MHz are input for study in these programs. Thus the predicted maximum usable frequencies may have up to 0.5 MHz systematic bias on the low side, since the actual MUF would be less than the lowest (integer) frequency for which no propagation is predicted by the raytrace scheme, but possibly higher than the highest one found supported.

(5) The "high-ray" calculation option in AMBCOM is chosen (HIRAY(I) = 0.6), permitting identification of possible high-angle rays on a given path.

(6) In AMBCOM, take-off angles from 0 to 45 degrees are considered, with one degree increments between angles.

A map showing the paths studied is given in Figure 1, and a summary of geographical, temporal, and solar parameters pertaining to the data is given in Table 2. We note that this study includes a short path for which most of the modes should be 1-hop, and two intermediate length paths for which composite mode propagation can be important. The short path data is at high sunspot number, while the longer path data is at low sunspot number. One of the longer paths can be considered a trans-auroral path while the other can be considered a trans-polar path (Folkestad [3]).

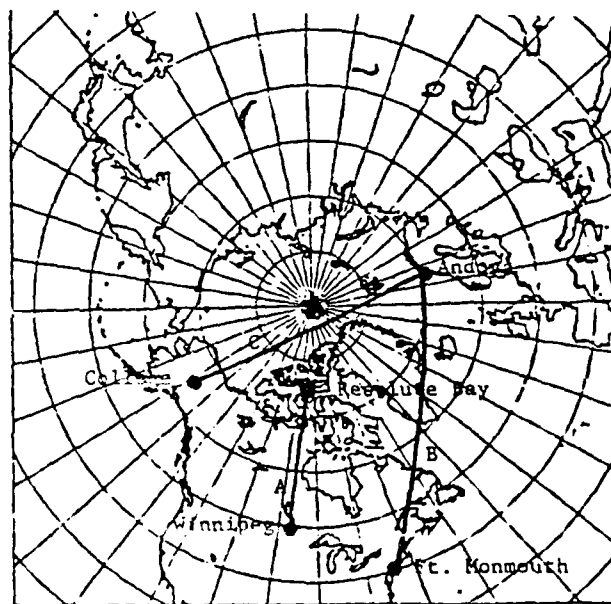


FIGURE 1 - WORLD POLAR PROJECTION SHOWING GREAT CIRCLE COURSES OF PATHS STUDIED

TABLE 2				
PATH ID	YEAR	ENDPOINT LAT (N+) LONG (W+)	GREAT CIRCLE DISTANCE (KM)	APPROX. MO. MED SUNSPOT NUMBER
A	1959	+49.5,+97.1 +74.7,+94.9	2799	140
B	1964	+69.1,-15.4 +40.3,+74.1	5853	10-20
C	1964	+69.1,-15.4 +64.9,+148.	5060	"

Section 3 - RESULTS OF THE STUDY

In Figures 2a-12b are presented the MUF predictions deduced from the two programs for each path-month studied, along with an identification (below the universal time, or UT, axis) of the mode which determines the MUF, and its corresponding total path loss in dB, for every two hours of UT. The results for the two programs are arranged side-by-side, the figure numbered with "a" corresponding to RADAR C predictions, and that numbered with "b" corresponding to AMBCOM predictions. The notation for modes used is explained in Davies [13]. A minus sign indicates a perigee ray, and a "v" indicates topside reflection. Thus, "E -F2" indicates a 2-hop perigee ray which reflects off of the E layer, intersects the earth, then reflects off of the F2 layer, reaching the receive site at an altitude of not more than 100 km (c.f. Section 1). Likewise, "F2 vF1 F2" indicates a mode which reflects off of the F2 layer, then off of the top-side of the F1 layer, then again off the F2 layer. An "H" refers to a high-angle mode.

SECTION 4 - DISCUSSION

The major features of the comparisons can be summarized as follows:

(1) For the Winnipeg-Resolute Bay path (Figures 2-4), both programs show the 1-hop F2 mode as the principal mode of propagation. Both programs predict the large observed diurnal variation in the winter season at high sunspot number, although AMBCOM has a bias on the high side for the diurnal peak.

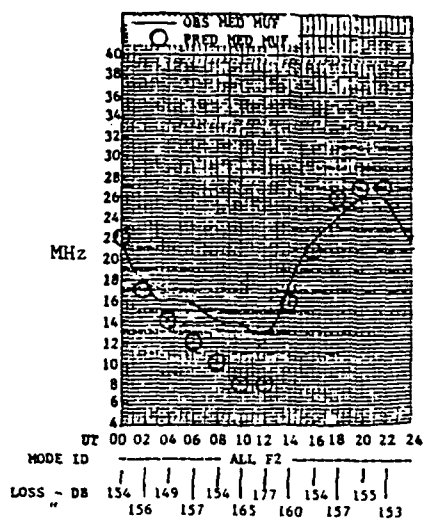


FIGURE 2a- RADAR C PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, OCTOBER, 1959

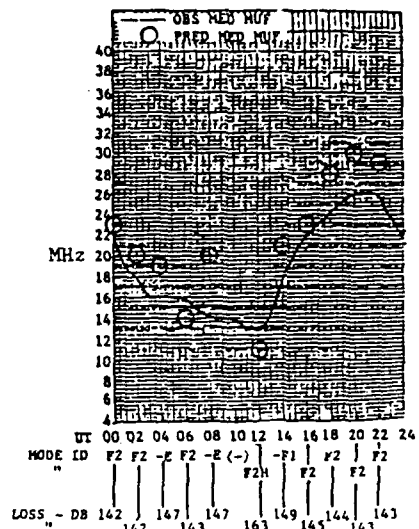


FIGURE 2b- AMBCOM PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, OCT., 1959

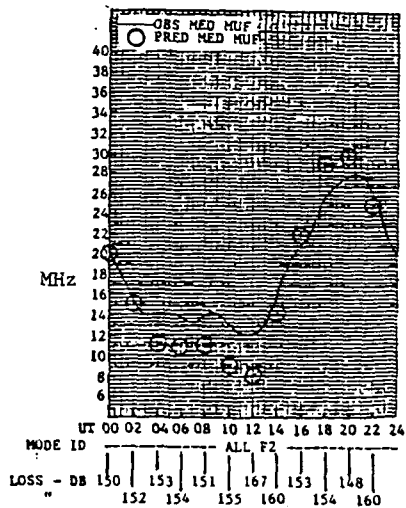


FIGURE 3a- RADAR C PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, NOV., 1959

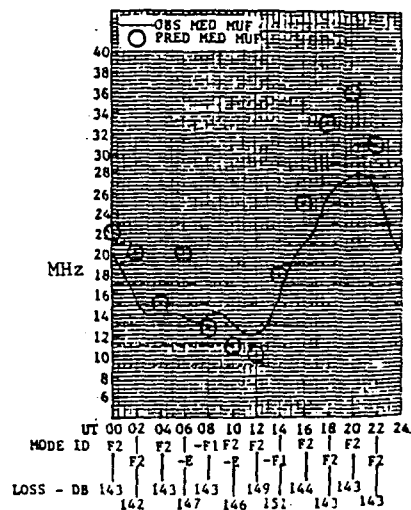


FIGURE 3b- AMBCOM PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, NOV., 1959

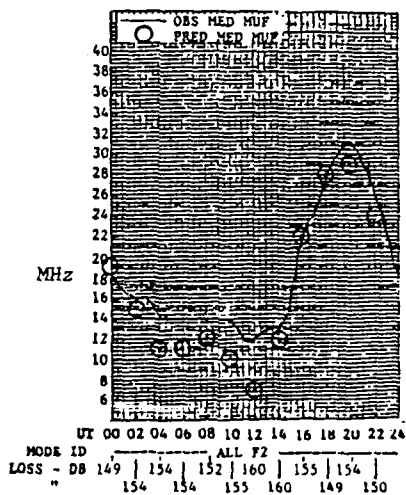


FIGURE 4a- RADAR C PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, DEC., 1959

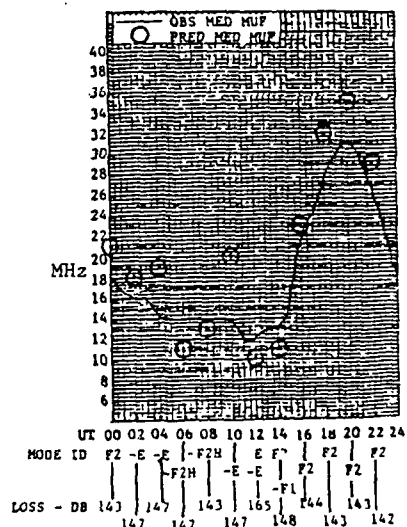


FIGURE 4b- AMBCOM PREDICTION FOR WINNIPEG-RESOLUTE BAY PATH, DEC., 1959

(2) For the Andoya-Ft. Monmouth path (Figures 5-8), AMBCOM predicts closer to the data than RADAR C, the latter having in general a low bias for the MUF. The modes found by AMBCOM for the MUF in this case involve several cases of composite modes, perigee modes, and high rays.

(3) On the Andoya-College path (Figures 9-12), AMBCOM shows a significant improvement in MUF prediction over RADAR C (which is generally 5-10 MHz too low), with composite and perigee modes playing an important part. There are several cases in which 2- and 3-hop modes involving a combination of E and F2 layer reflections, as well as perigee modes, determine the MUF. Although AMBCOM is a significant improvement over RADAR C in this case, there is room for more improvement, as AMBCOM is still biased on the low side of observed median MUF's.

(4) As a by-product of the fact that AMBCOM generally finds higher MUF values (which are closer to the observed values) than RADAR C, AMBCOM also shows lower path losses for these higher frequencies, so that the required power on certain paths may be significantly lower than that predicted by RADAR C.

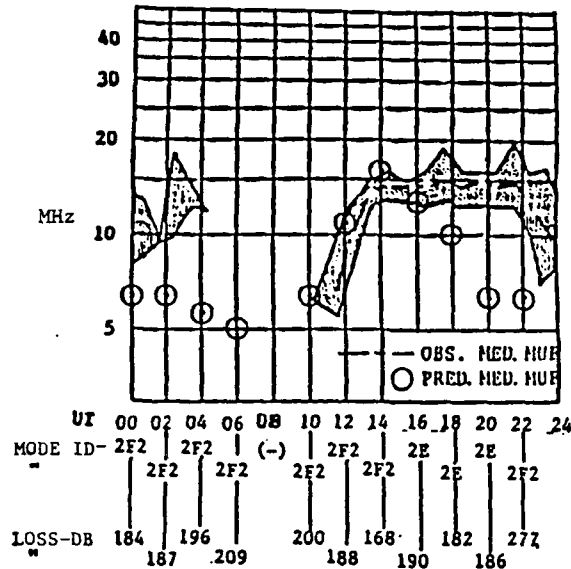


FIGURE 5a - RADAR C PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, JANUARY 1964

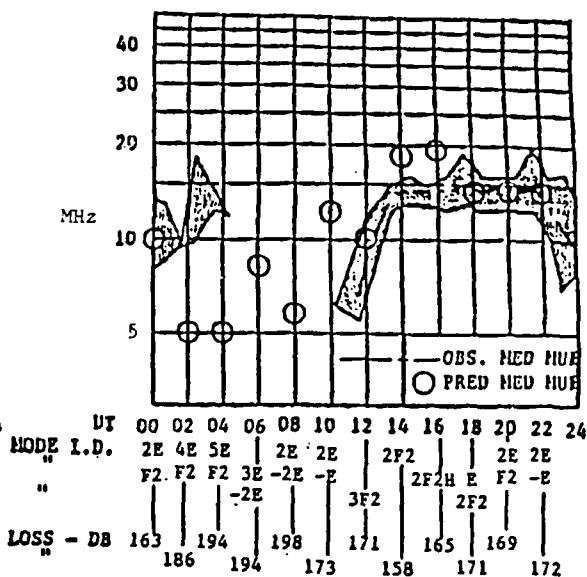


FIGURE 5b - AMBCOM PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, JANUARY, 1964

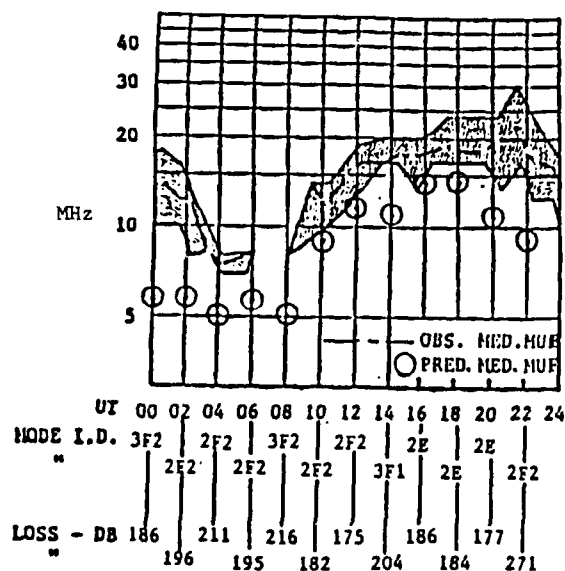


FIGURE 6a- RADAR C PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, MARCH, 1964

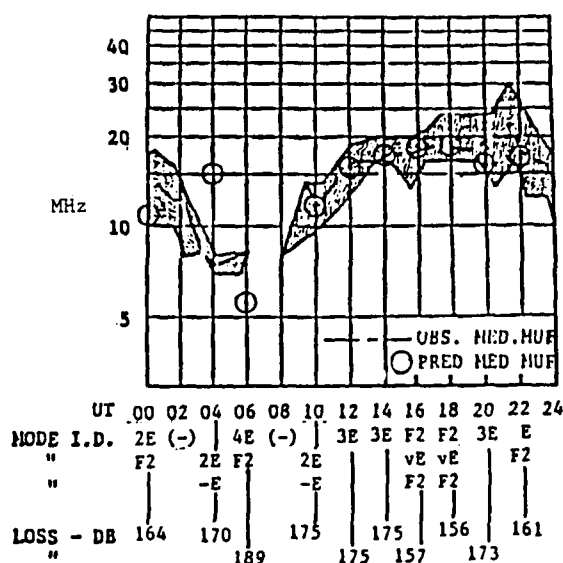


FIGURE 6b- AMBCOM PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, MARCH, 1964

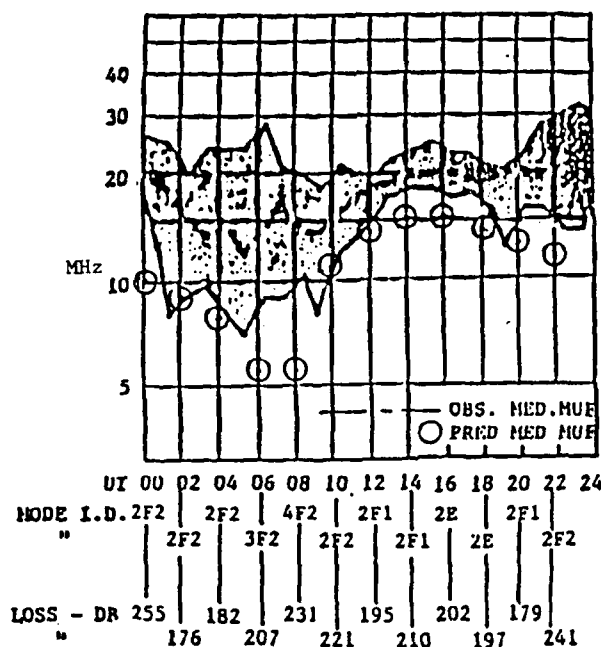


FIGURE 7a - RADAR C PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, MAY, 1964

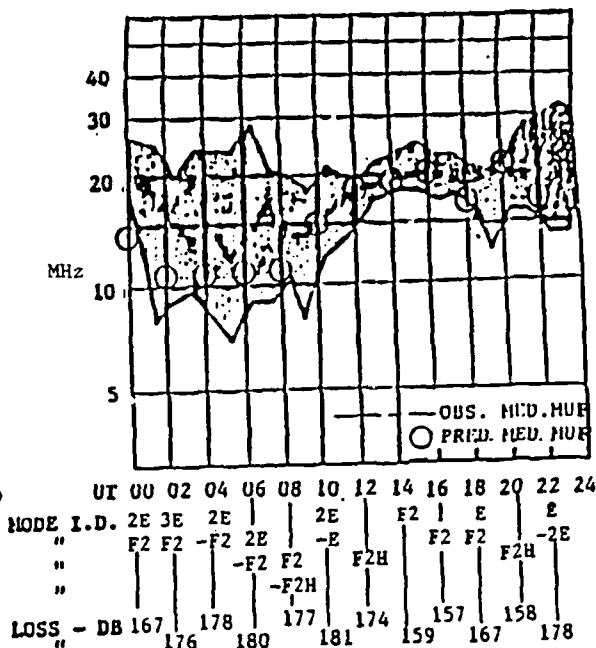


FIGURE 7b - AMBCOM PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, MAY, 1964

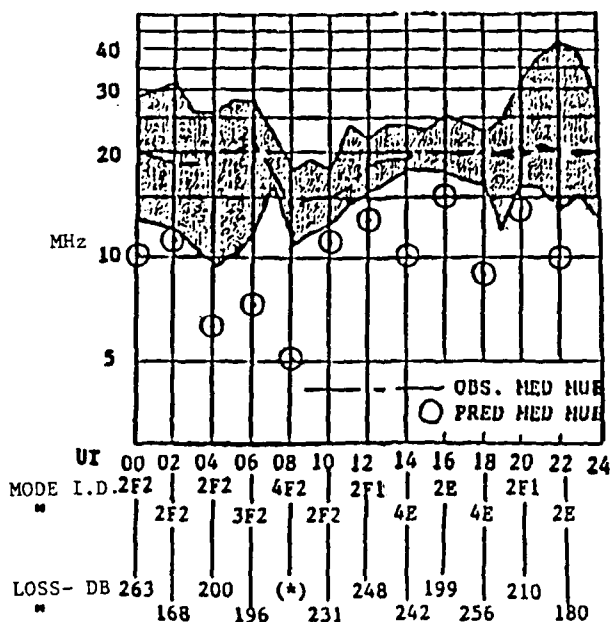


FIGURE 8a - RADAR C PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, JULY, 1964

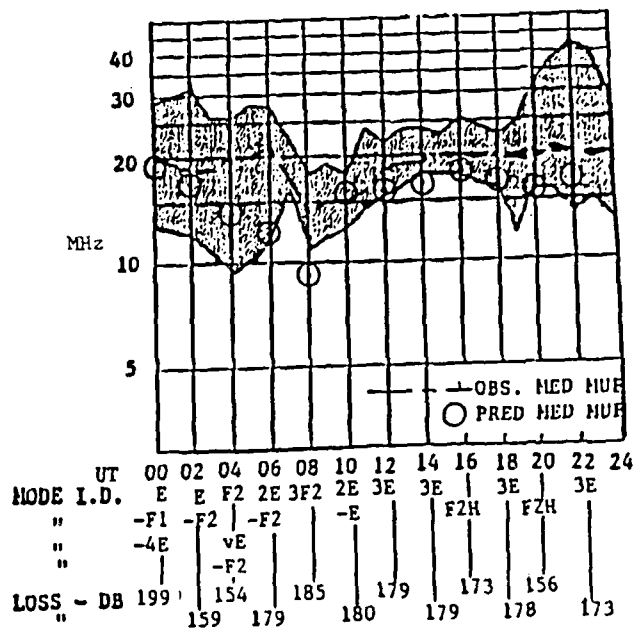


FIGURE 8b - AMBCOM PREDICTION FOR ANDOYA-FT. MONMOUTH PATH, JULY, 1964

For added insight, a comparison of the calculated values of E- and F2-layer critical frequencies and heights of the layer maxima for the two programs on the Andoya-College path at 6 hour intervals is plotted in Figure 13. (E-layer maximum height is a constant 130 km in RADAR C and 115 km in AMBCOM). This figure shows that the ionospheric parameters on the Andoya-College path differ for the most part by only a few percent between the two programs so that one can

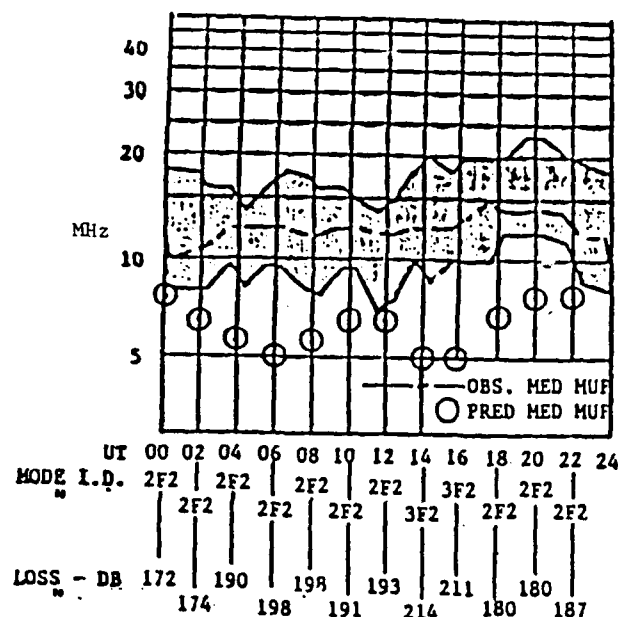


FIGURE 9a - RADAR C PREDICTION FOR ANDOYA-COLLEGE PATH, JANUARY, 1964

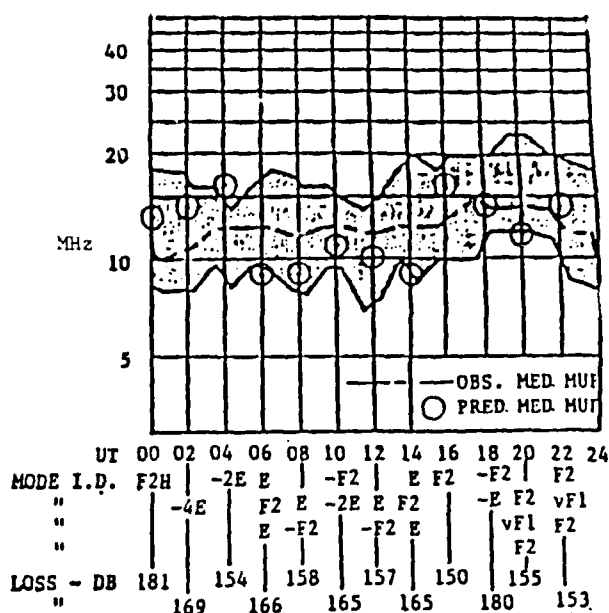


FIGURE 9b - AMBCOM PREDICTION FOR ANDOYA-COLLEGE PATH, JANUARY, 1964

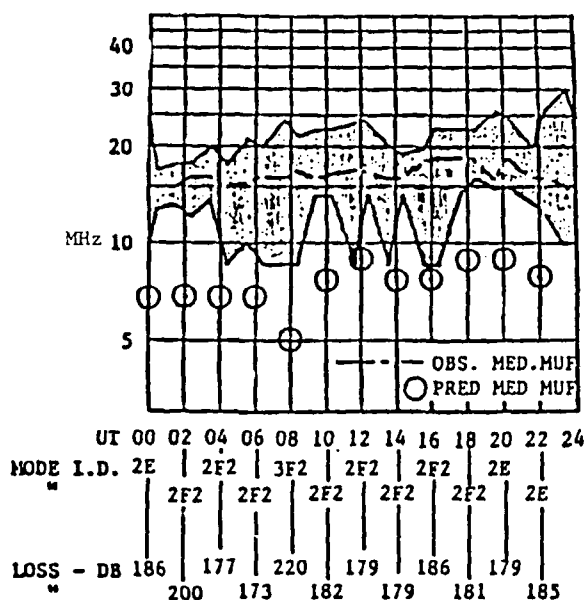


FIGURE 10a- RADAR C PREDICTION FOR ANDOYA-COLLEGE PATH, MARCH, 1964

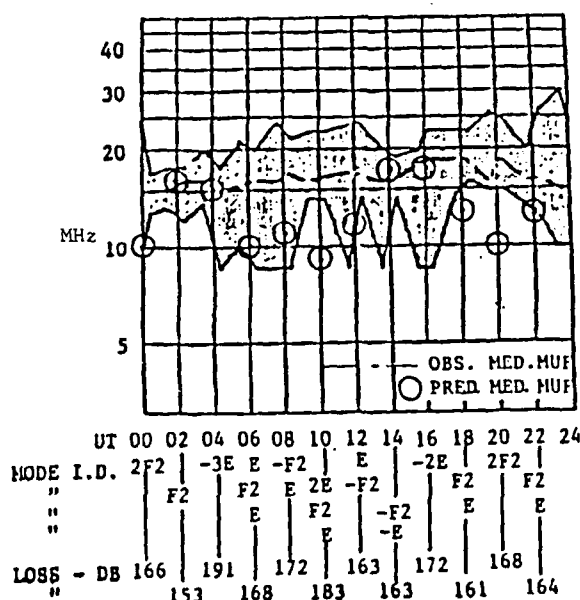


FIGURE 10b- AMBCOM PREDICTION FOR ANDOYA-COLLEGE PATH, MARCH, 1964

conclude that the radical improvement in MUF prediction of AMBCOM over RADAR C for this path is not due mainly to the values of ionospheric parameters used. Rather, based on the types of modes found to constitute the MUF in AMBCOM, it is to be concluded that the more accurate, and physically more realistic raytracing in AMBCOM, combined with many more control points than used in RADAR C, are the main causes for the significant improvement in MUF prediction in AMBCOM compared to RADAR C. These capabilities (c.f. the discussion, item (3) of Section 1) allow for the consideration of tilts and composite and perigee modes, which is not possible in

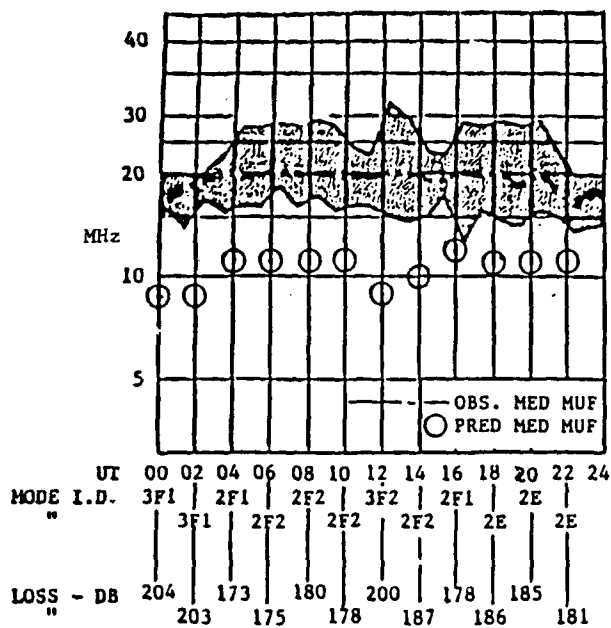


FIGURE 11a- RADAR C PREDICTION FOR ANDOYA-COLLEGE PATH, MAY, 1964

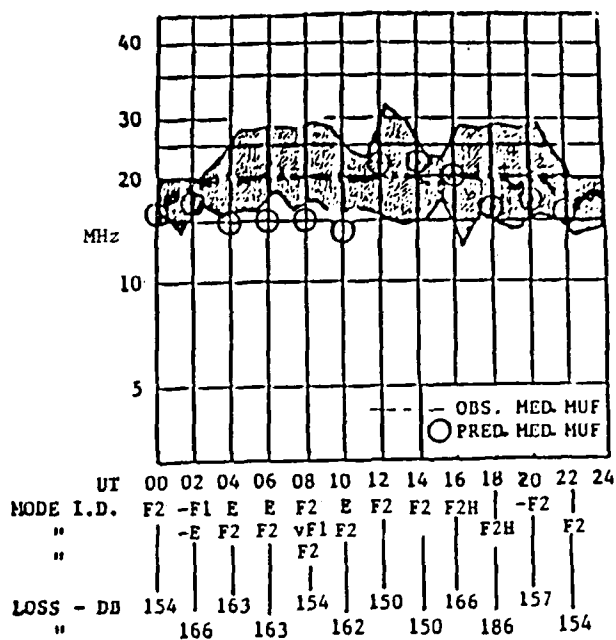


FIGURE 11b- AMBCOM PREDICTION FOR ANDOYA-COLLEGE PATH, MAY, 1964

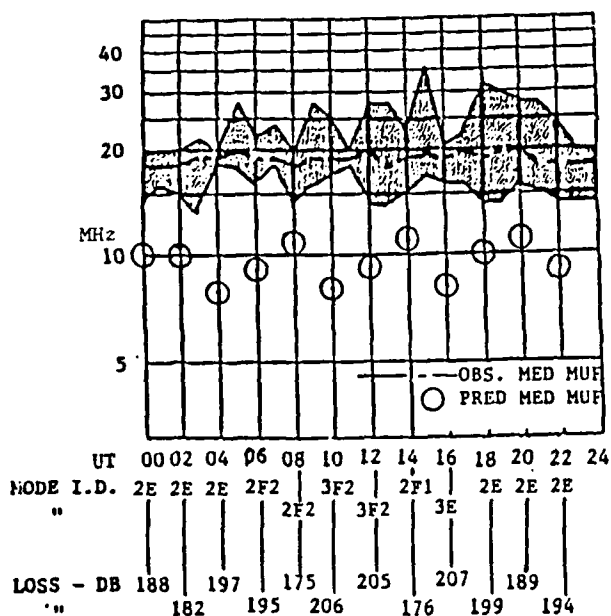


FIGURE 12a- RADAR C PREDICTION FOR ANDOYA-COLLEGE PATH, JULY, 1964

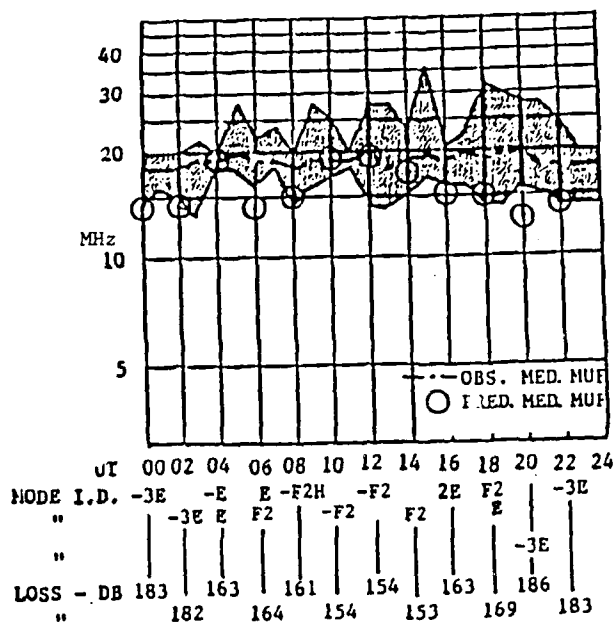


FIGURE 12b- AMBCOM PREDICTION FOR ANDOYA-COLLEGE PATH, JULY, 1964

RADAR C. This conclusion is consistent with the study by Paul [14] of the importance of horizontal gradients in electron density in the ionosphere even at mid-latitudes.

A similar comparison of ionospheric parameters for the two programs for the trans-auroral path (Andoya-Ft. Monmouth) is shown in Figure 14. This figure supports the conclusion that for this path, in addition to the effects of improved raytracing, differences in ionospheric modelling (especially for the E-layer critical frequency), are also significant causes for the improved predictability.

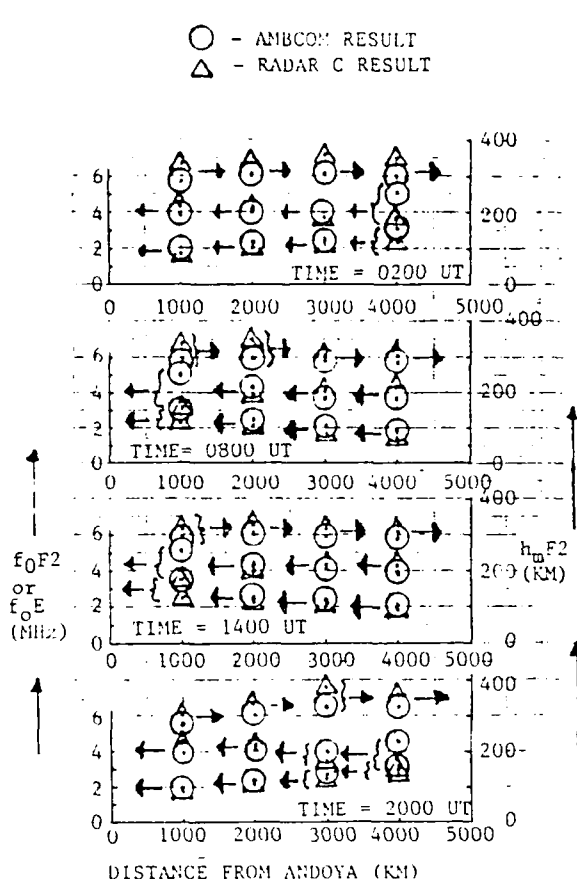


FIGURE 13 - COMPARISON OF f_0F_2 , f_0E , and h'_F2 CALCULATED BY RADAR C AND AMBCOM FOR ANDOY-COLLEGE PATH, JULY, 1964

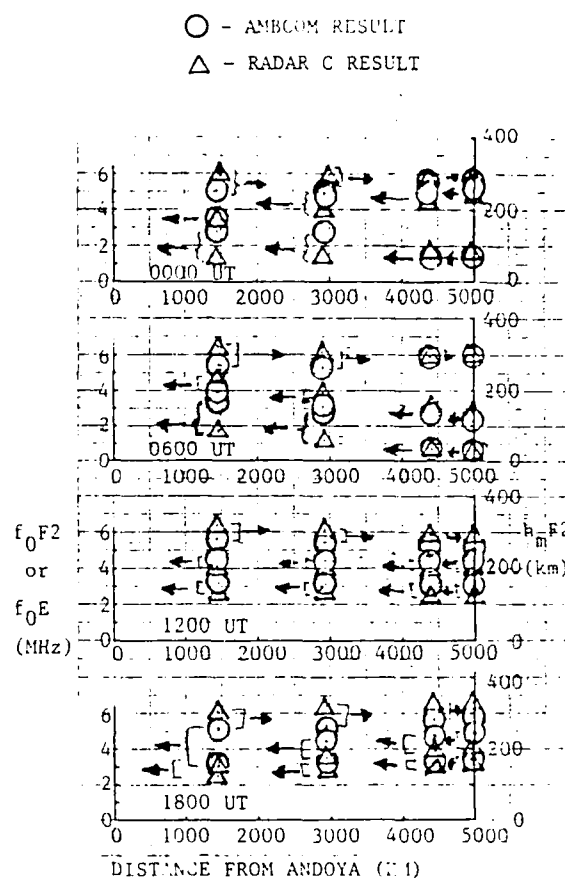


FIGURE 14 - COMPARISON OF f_0F_2 , f_0E , and h'_F2 CALCULATED BY RADAR C AND AMBCOM FOR ANDOYA-FT. MONMOUTH PATH, JULY, 1964

SECTION 5 - CONCLUSIONS AND LINES FOR FURTHER RESEARCH

AMBCOM in general performs better as a MUF predictor than RADAR C on the high-latitude paths studied, the latter having a significant low bias for MUF on the trans-polar and trans-auroral paths studied, although AMBCOM has somewhat of a high bias on the Winnipeg-Resolute Bay path. Based on the discussion in Section 4, the improved performance of AMBCOM over RADAR C, at least on the trans-polar path between Andoya and College is primarily due to its more physically realistic raytracing scheme. On this path, composite and perigee modes often determine the MUF. Further improvement for the prediction of this high-latitude path and for the Andoya-Ft. Monmouth path is, however, needed.

For future research, a more complete test of AMBCOM is desirable, using a larger database with a wide variety of path-months. This will identify possible improvements which can be made to the raytrace scheme of AMBCOM. Since this study shows that accurate raytracing is important on the high-latitudes paths studied, it is reasonable to hypothesize that incorporation of a three-dimensional ray-tracing routine (e.g., Jones [15]) into AMBCOM in place of the present one will reveal other, higher frequency modes of propagation not propagating on great circle paths, leading to further improvement in predictive capacity. The present version of AMBCOM does not include non-great circle (NGC) propagation, as was noted in Section 1. We note the discussion by Hunsucker and Bates [16] of the importance of NGC modes in high-latitude propagation, and the fact that this may also be of significance at lower latitudes.

Observationally, since few ionosondes have been operated in polar regions, it is to be expected that a program of vertical incidence ionospheric critical frequency measurements in the polar region will improve our capacity to model HF propagation in this important region of the world.

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